... tend to separate the bridge into ... along the lines *ab* and *st*. Peyronnet considered these points to occur in a semi-circular arch at intervals of about sixty degrees from the vertex, and this estimate may be used when mathematical accuracy is not pretended to.* In fact their position varies according to the depths of the arches and the loads upon them, and these being assumed, it can only be ascertained by approximation. The object is to determine such point in the intrados, as, taken for a fulcrum, would give the greatest possible value to the horizontal thrust.

Let the area *abcd* be equal to W, *af*, the horizontal distance of its centre of gravity from the vertical line *ob*, equal to D, the versed sine *de* plus, half the height of the key-stone, equal to X, then, $W \times D = X \times S$ and $\frac{W \times D}{X} = S$ an area proportional to the greatest horizontal stress.

Upon *ae* set off *ag*, equal to the height of the key-stone, and draw the vertical line *gh*, intersecting the tangent to the curve in *h*, *ah* is the thickness (nearly) which should be given to the arch at the points *a* and *s* upon the first foregoing hypothesis of what is proper. Draw *ik* parallel to the tangent intersecting the vertical line in *k*, *ak* is the vertical depth of the arch upon the second hypothesis. The bridge being supposed equally loaded on both sides, the curve of equilibrium would be alike on both sides, and would somewhat resemble the dotted sketch, *al*. The assumed extraneous load being supposed removed from one side of the bridge, a new condition of equilibrium arises, the most unfavourable possible under the assumptions, and ordinates as *mn*, *op*, *qr*, of the new curve, are to be found. It will be sufficient to shew a method of determining one of them, as any other may be found in the same way. When a lever, supported by a fulcrum, is in equilibrio, with two forces acting at different distances from the fulcrum, the sum of the forces is to the sum of the distances as the greatest force to the greatest distance, and as the least force to the least distance. Let *o* be the fulcum in question; *op* is the greatest distance, and *os* the least; and let P be the area proportional to the horizontal thrust of the mass, *stud*, obtained by taking *s* as a centre of motion; the force acting at the vertex *l*, tending to destroy the equilibrium, is as the difference between S and P, the greatest force that, acting against *s*, is as P, the sum of the two as S, and S : P : : (*sp*) : (*op*) the ordinate sought for.

$$\frac{P \times (sp)}{S} = (op)$$

In the example the radius was made unity, the depth of the key-stones one-tenth, the depth of the permanent load over the crown one-twentieth. The point of fracture, *a*, was taken at fifty-four degrees from the vertex; and from these data the depth (*uc*) of the maximum extraneous load was determined to be ·10707, the ordinates, *mn*, *op*, *qr*, corresponding to 27, 54, and 90 degrees of the intrados, as laid down to the scale. The curve of equilibrium never approaches the extrados of the arch; and it is therefore evident that it would be a very good arch under the load allowed for, provided it were constructed of materials capable of resisting the pressures generated, without much yielding. The surface proportional to the horizontal thrust was found to be ·22375, which, divided by one-tenth, the depth of the key-stones, gives 2·2375 for the height of the columnar measure of the horizontal thrust. The above linear dimensions being multiplied by 30 feet, give for the span of a bridge 60 feet, depth of the key-stones, 3 feet, height of the permanent load above them, 1 foot 6 inches, depth of the variable load, 3·21 feet,† and height of the columnar measure, 67·1 feet; this last, multiplied by the weight of a cubic foot of the material (let it be brickwork, 117 pounds) gives 7850 pounds as the pressure per square foot. Now it is probable, that a brick wall 3 feet thick, 67 feet high, constructed on a base of well consolidated brickwork, would have but little power to compress that base; it may therefore be reasonably concluded, that such an arch, constructed of good brickwork, would be sufficiently secure. The modulus of fracture of good brickwork, that is to say, the height of a column of it, equal in weight to the crushing weight, has been estimated at 600 or 700 feet: if this be so, then the arch would be about ten times stronger than necessary to resist actual crushing. If the arch were made of the same thickness throughout, as is usually the case with arches of brickwork, it would be intersected by the curve of equilibrium near the point *o*, and it would be advisable to reinforce it with sound materials, at least as high as *oS*, to make it safe with the load provided for. If a greater load were imposed, fracture would be very likely to occur at that point, and at the lowest point (*d*) of the arch at the vertex. The strain should be estimated according to the resultant pressure, not according to the horizontal thrust as before. It has been assumed, that an arch has to sustain the whole burden. This is not actually the case: though the spandrils were filled in with loose gravel, they would contribute some aid to restrain the horizontal thrust. As was before stated, the theory merely teaches what is necessary under certain assumed circumstances; what those circumstances shall be must depend upon the judgment of the calculator; and if he is cautious, he will take care that his allowances are ample. It has also been assumed, that the abutment walls are immovable. If an arch of the kind be made very thin, the curve of equilibrium will intersect it very near the crown; and it will owe its stability almost wholly to the horizontal resistance of the spandrils. The arch of an equilibrated bridge is first supposed to be attenuated to the smallest thickness; and the curve of equilibrium is made to agree with it. The thicker the arch is actually made, the greater extraneous load will the bridge bear, without the new curve of equilibrium intersecting the arch.

Perhaps the most important result of theory is the disclosure of the enormous pressures which are generated; surely if practical men were aware of this, not so many failures of arches would occur, many no doubt, arising from the centres being struck before the materials are sufficiently consolidated to resist the stress upon them.

The columnar measure of the horizontal stress for semi-circular and semi-elliptical bridges, with horizontal roadways, may be determined sufficiently near for practical purposes by the following formula, deduced upon the supposition that the points of fracture, *v*, *w*, are midway between the springings and the crowns, and that the area included between them are parabolic.

Let $r$ = half the rise;
$s$ = half the span, corresponding to half the rise;
$k$ = the height of the key-stone;
$d$ = the whole depth of the bridge, and load over the vertex.

$$\frac{s^2 \times (r + 6d)}{12k \times \left(r + \frac{k}{2}\right)} = \text{The columnar measure.}$$

Applied to the preceding example it gives,—

$$\frac{26 \times 26 \times (15 + 6 \times 7{\cdot}71)}{12 \times 3 \times \left(15 + \frac{3}{2}\right)} = \frac{41411}{594} = 69{\cdot}7 \text{ feet.}$$

Applied to segmental arches the arcs not exceeding 120 degrees, $r$ = the whole rise, $s$ = half the span, $k$ and $d$ as before.

Let $s$=65 ft., $h$=25 ft., $k$=3·5 ft., $d$=8 ft.

$$\frac{65 \times 65 \times (25 + 6 \times 8)}{12 \times 3{\cdot}5 \times \left(25 + \frac{3{\cdot}5}{2}\right)} = \frac{308425}{1123{\cdot}5} = 265{\cdot}6 \text{ feet.}$$

The modulus of fracture of Aberdeen granite has been estimated at 9,581 feet; of Portland stone, 4,337 feet; of brick, 621—858 feet. The reader may make his own inferences.

In conclusion, it may be remarked, that when a bridge has been designed, of the form best adapted to the local circumstances, whatever that form may be, the above particulars may be calculated (the liberal assumptions which must be made shew that there is no occasion for minute accuracy) by the ordinary rules of mensuration and arithmetic, with as much facility as an estimate of its cost. It is supposed, of course, that the calculator is acquainted with the first elements of mechanics, such as are necessary for the understanding of this paper. J. P. W.

* The points of fracture of the arch of the bridge of Nogent-sur-Seine (a false ellipse), were observed by him to be very nearly a sixth of the whole intrados away from the springings. The points of fracture of a semi-circular arch being known, those of an elliptical one may be determined from them if all the corresponding heights of the arches and loads have the same proportion; they are distant from the abutments by the same proportional parts of the span in both. The same observation applies to all forms of bridges which may be deduced from each other by lengthening or shortening the dimensions one way, such as the form of the equilibrated bridge with a horizontal roadway which may be obtained from the dimensions of the common catenary. The method of obtaining this form of arch directly is ably treated in THE BUILDER page 506.

† Another trial and inspection of the Rouen and Havre line was made last week by the engineers of the company; the whole line being run over by the engine and carriages, except the viaduct of Malaunay, which is now undergoing the extraordinary trial ordered by the Government engineers. This is to lay upon it, for one month, ballast of sufficient thickness to weigh three tons per square yard superficial, which might become four tons if the weather be very wet during the period of trial. It is now said, that should the line go successfully through this ordeal, it may then be opened throughout.—*Railway Times.*

## PRINCIPLES OF SYMMETRICAL BEAUTY.

SOCIETY OF ARTS.

DEC. 16th, Dr. Roget, Sec. R.S., Vice-President, in the chair. The secretary read an address from the council, which gave a retrospect of the proceedings of the past year, and the proposals of the council for the future. It stated that formerly the society, as is well known, stood alone as the great active, scientific, mechanical, and artistic circle of London, the Royal Society being the only other in any analogous position. That now, however, that great field is happily full of co-operating societies, each labouring on some one subject, formerly a mere dependant on its vast territory. That this removal from the parent society of so many branches has necessarily stripped it of many of its bright ornaments, but it appears to the council that far from being regarded as an evil, this multiplication of useful societies is a subject for congratulation, and should be regarded as one strong proof of its past usefulness. The council consider that the field on which the society might, with best effect, concentrate its future labours, as well as that which most properly belongs to it, is a department of the fine arts hitherto much neglected in this country, and which has been strongly approved of by H. R. H. Prince Albert, president of this society, viz., that of promoting high art in connection with the mechanical, for which our manufacturers are so justly celebrated.

After a communication "On the principles employed in the recent decorations of the great room," a paper was read "On the first principles of symmetrical beauty, and their application in certain branches of the art of design," by D. R. Hay, Esq. This paper commenced by stating that the first principles of symmetrical beauty originate in the powers of numbers, and that a means of applying the principle of numbers in the formation of plane figures, is afforded by the division of the circumference of the circle into 360 degrees, which degrees are again divisible and subdivisible by 60 into minutes, seconds, &c. Thus the abstract principle of harmony and proportion, in the relations of certain numbers to each other, becomes apparent and visible, in their application to the structure of geometrical figures, by means of the division of the circle. It then proceeded to show that to apply these degrees to rectilinear plane figures, each figure must be reduced to its primary element. That the triangle, which is half of the square, is the first and most simple of its class, and is the representative of No. 2; that the scaline triangle, which is half of the equilateral triangle, is, in like manner, the representative of No. 3; that the next scaline triangle which arises naturally in the series, is that which is half of one of the five isosceles triangles which form the pentagon, and is the representative of No. 5. We have, therefore, in the square, the equilateral triangle, and the pentagon, the primary elements of all symmetrical beauty as represented by plane figures, and evolving the operation of the harmonic numbers of 2, 3, and 5. Out of the primary rectilinear figures already referred to, arises a second class, as when an equilateral triangle is divided into two scaline triangles by a line drawn through one of its angles, and bisecting the opposite side, these scaline triangles if reunited by their hypothenuses instead of their longest sides, will form an oblong rectangle, every rectilinear figure having its corresponding curvilinear figure.—The paper concluded by attempting to show the operation of the principles of harmonic ratio in the formation of the mouldings of Grecian architecture, ornamental vases, and household utensils.